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THE USE OF QUASIGEOID IN LEVELING THROUGH TERRAIN OBSTACLES

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Abstract

In these paper are presented two ways of performing leveling through terrain obstacles. They use properties of the quasigeoid course with respect to the ellipsoid within a given area. The analysis of changes in quasigeoid to ellipsoid slope have been made on the basis of the national quasigeoid models, calculating the slope components ξ,η. This allows to present practical recommendations for location of intermediate benchmarks in the leveling methods through obstacles.

Keywords: *classical and satellite leveling, leveling through terrain obstacles*

1. Introduction

Differential (geometric) leveling is a method allowing for the establishment of differences in elevation between two or more points on the basis of position of the horizontal axis of a leveling instrument with respect to vertically placed leveling rods. Implementation of such a measurement method is made difficult in case of a large height difference between endpoints or some terrain obstacles e.g., broad rivers. Routing the geometric leveling, maintaining the principle of leveling from the midpoint and the lengths of lines of sight not exceeding 50 m, avoiding the obstacle is not always possible. A terrain obstacle, dividing a leveling network into parts, causes its heterogeneous construction. Leveling connections made by bridges or viaducts may be insufficient, for example due to a small number of them, distances between them or the unfavorable location relative to the measured leveling network.

In order to limit the impact of these difficulties, methods of leveling through field obstacles were developed. These methods use, for example, special target plates that complement the leveling set, or a measurement by means of a pair of levels (Niwelacja precyzyjna, 1993). Another solution is the use of trigonometric leveling. Synchronous angular observations performed in this method significantly reduce the adverse effect of vertical refraction (Schödlbauer et al., 1993; Walo, 1994), increasing the accuracy of measured height differences.

It is also possible to determine a given height difference indirectly by means of GNSS measurements, taking into account the quasigeoid (or geoid) course. GNSS measurement methods using information about the course of this surface with respect to an ellipsoid seem to be an alternative to the aforementioned leveling methods. Below there are presented two ways of performing leveling through terrain obstacles. They use properties of the quasigeoid course with respect to the ellipsoid within a given area.

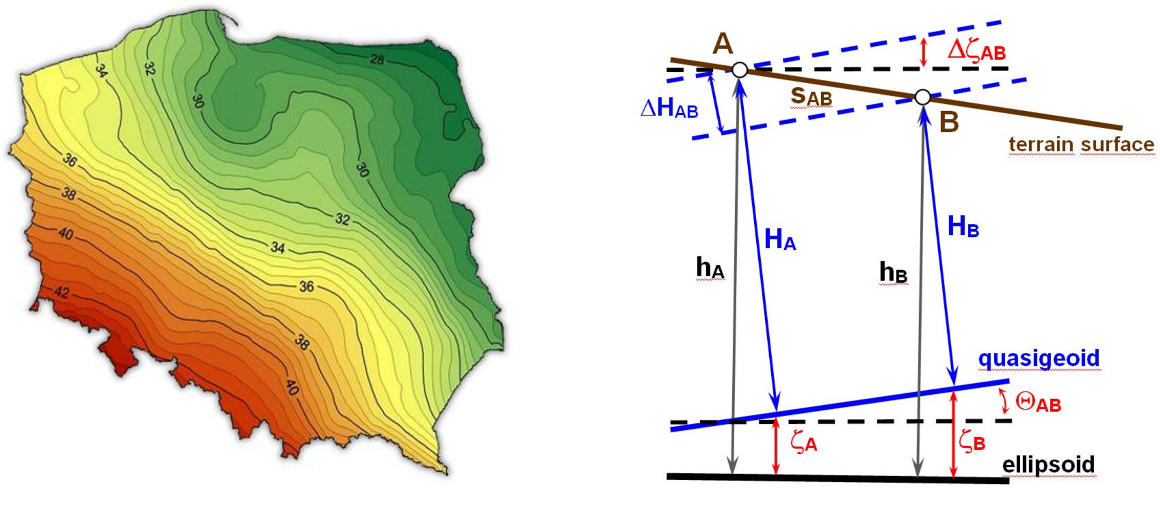
2. Basic relations between differential leveling and satellite leveling

The relationship between the difference of ellipsoidal heights Δh and normal heights ΔH between points A and B is given by the equation:

$$\Delta H_{AB} = \Delta h_{AB} - \Delta \zeta_{AB} \tag{1}$$

where $\Delta\zeta_{AB}$ is an increment of height anomaly between points A and B. The value of such an increment can be derived from a suitable quasigeoid model. Models used by surveyors in Poland include "Geoida niwelacyjna 2001" (Pażus et al., 2002) and the latest model "Geoidpol-2008CN" (Geoidpol 2008). In the second model, it is possible to calculate ΔH_{AB} basing on given values of Δh_{AB} and ellipsoidal coordinates φ, λ of endpoints of the leveling segment AB.

The course of quasigeoid with respect to the ellipsoid in Poland varies. Quasigeoid is located above the ellipsoid at an average height of 35 m (Fig. 1a). In most areas of the country, the two surfaces are inclined at an angle Θ reaching several to over a dozen arcseconds (Figure 1b). The direction of this inclination is roughly north-east. On a quasigeoid map, however, there are areas where the inclination and its direction differ significantly from the mean values (Figure 1a). Such areas include areas of north-eastern Poland, in the belt from Torun in the center of the country to Lublin in the east.



a) b)

Fig. 1. A course of quasigeoid with respect to the ellipsoid:
a) isolines of height anomaly within Poland (Geoida IGiK),
b) relation between ellipsoidal heights (h) and normal heights (H), and the increment of height anomaly $\Delta\zeta$

The increment of height anomaly $\Delta\zeta_{AB}$ given in equation (1) that characterizes variation in quasigeoid shape is dependent on the azimuth of the segment AB (α_{AB}) and is proportional to the distance S_{AB} between the points (Schödlbauer et al., 1993). For small areas, this increment may be represented by the formula (2):

$$\Delta\zeta_{AB} = S_{AB} \cdot (\xi \cos\alpha_{AB} + \eta \sin\alpha_{AB}) = S_{AB} \cdot \Theta_{AB} \tag{2}$$

where Θ_{AB} is the quasigeoid to ellipsoid slope in the direction of AB, and ξ and η are the components of the slope in the meridian and prime vertical (latitudinal) directions respectively[1]. An illustration of quasigeoid to ellipsoid slope variation depending on the azimuth is shown in Figure 2.

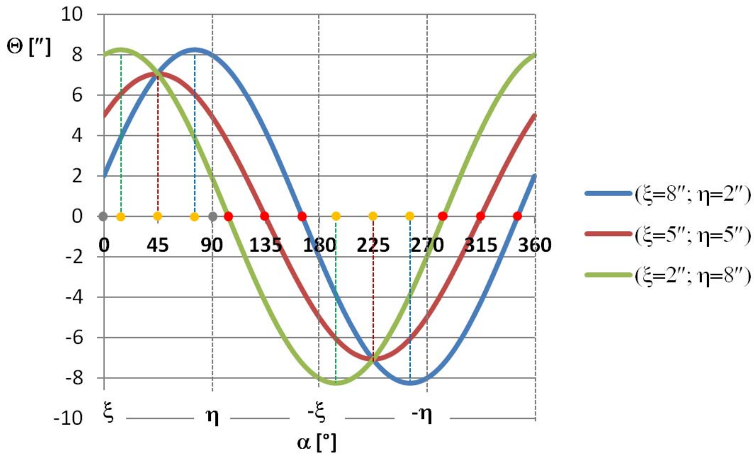


Fig. 2. Quasigeoid to ellipsoid slope in characteristic directions

Graphs were made for cases of varied quasigeoid to ellipsoid slope. The first graph (blue) shows the case where the quasigeoid is inclined more to the north (ξ = 8″) than to the east (η = 2″). The third graph (green) presents the opposite situation (ξ = 2″, η = 8″). The second graph (burgundy) shows the quasigeoid to ellipsoid slope precisely in the north-east direction ($\xi = \eta$ = 5″). Yellow dots on the horizontal axis indicate azimuths in which the inclination of quasigeoid to ellipsoid is maximum and the red ones indicate azimuths of zero inclination. Isolines, by means of which the modeled shape of quasigeoid with respect to ellipsoid is often presented, run along the azimuths of zero slope (Geoida IGiK).

3. Methods of leveling measurements through terrain obstacles

Two methods of leveling through terrain obstacles with the use of GNSS proposed below are based on an assumption that changes in quasigeoid course with respect to the ellipsoid are slight and are of linear character. Moreover, it has been assumed that points are placed close enough so that the ellipsoid and quasigeoid may be approximated by planes in their vicinity. Assumed accuracy of leveling determines the way of performing the GNSS measurement.

[1] Direction of the prime vertical roughly corresponds to the direction of the parallel.

3.1. Method I – „benchmarks in isolines of quasigeoid”

On the basis of well-known theorem from basics of geometry, it can be stated that if two planes intersect, then on each of them there exists one and only one straight line passing through the given point and parallel to the second plane. An illustration of this theorem is the conclusion coming from Figure 2, showing that for any values of ξ and η one may find a direction α_0 for which the slope Θ will be zero. Such an azimuth α_0 may be computed after transforming relation (2) to the form (3) assuming that $\Theta=0$:

$$\alpha_0 = -arctg\frac{\xi}{\eta} \tag{3}$$

On the direction α_0, according to the relation (1) an increment of height anomaly $\Delta\zeta$ will be zero. It means that ellipsoidal height differences Δh_{AB} and their normal counterparts ΔH_{AB} will be equal and leveling measurement may be replaced with GNSS measurement (assuming appropriate accuracy). This is a special case of satellite leveling along a certain direction. A sketch of this kind of usage of quasigeoid in leveling through field obstacles is shown in Fig. 3.

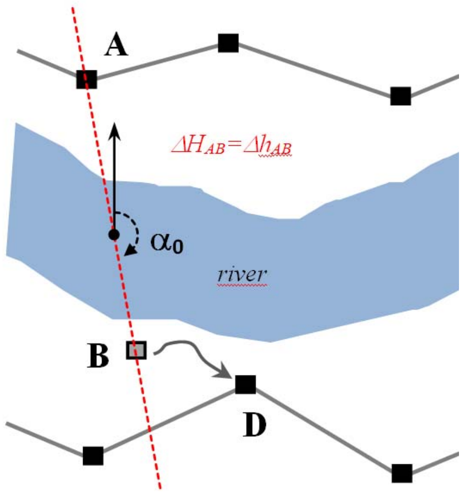


Fig. 3. Outline of the use of information on the course of quasigeoid in leveling through field obstacles for benchmarks located in the isolines (contour lines) of height anomaly

Benchmarks of a leveling network are located on both sides of the river. The direction α_0 passing through the selected benchmark A is indicated by a red, dashed line. Values ξ and η necessary for its calculation were derived from the available quasigeoid model. Leveling through a terrain obstacle, the position of an auxiliary benchmark B is set at the opposite bank of the river, in the direction α_0. This benchmark does not have to be stabilized, it should only be connected to the remaining benchmarks of the leveling network (point D in Fig. 3). For new or upgraded leveling networks, location of benchmarks A and B on both sides of the river may be designed so that they are located in the isolines (contour lines) of the height anomaly.

GNSS measurements should be performed on benchmarks A and B, and their result in the form of a height difference Δh_{AB} will be equal to ΔH_{AB} that would have been obtained by classical leveling. It should be emphasized that the GNSS measurement method should be selected in a way to provide the expected leveling accuracy in a given vertical network.

The method of „benchmarks in isolines of quasigeoid" relies on location of benchmarks according to a determined azimuth and measurement of ellipsoidal height differences by means of GNSS technique. Hence, the question arises as to how accurately the position of isolines should be determined by specifying its azimuth at a given point? Transformation of formula (2), followed by its differentiation, leads to the relation (4) for calculating the azimuth error m_α depending on the error of height anomaly $m_{\Delta\zeta}$:

$$m_\alpha = \frac{m_{\Delta\zeta}}{S \cdot (\eta \cos\alpha - \xi \sin\alpha)} \tag{4}$$

The minimum value of m_α will be for the azimuth α = 135° which roughly corresponds to the direction of isolines of the quasigeoid course in Poland. Assuming also, that the components ξ, η will be equal and reach maximum values occurring in Poland (i.e., $\xi=\eta=12''$), the error m_α may be estimated from the following working formula:

$$m_\alpha = \frac{\rho''}{12''\sqrt{2}} \frac{m_{\Delta\zeta}}{S} \tag{5}$$

The $m_{\Delta\zeta}/S$ fraction found therein may be identified with an error describing the relative accuracy of the vertical network. Hence, for the primary vertical network ($m_{\Delta\zeta}/S = 1.5 \cdot 10^{-6}$) one obtains m_α = 1.0°, and for the detailed vertical network ($m_{\Delta\zeta}/S = 4 \cdot 10^{-6}$) one obtains 2.7°. It is relatively easy to stake out the direction with such accuracy. It should be noted that the above calculations were made for extreme slope values ξ, η. Under different conditions the error m_α may be larger and in order to achieve assumed accuracy one may stake out such a direction with less precision. For example, in the region of Krakow where deflection components equal to $\xi=2.8''$ i $\eta=8.6''$ such an azimuth may be determined nearly twice less accurately.

3.2. Method II - uniform course of the quasigeoid

Analysis of equation (2) leads to the conclusion of possibility of using the proportionality of quasigeoid to ellipsoid slope increments between a benchmark A located in the leveling line and an arbitrary selected working benchmark B (offset $\Delta\zeta_{AB}$ to distance S_{AB}). On a such specified direction AB, on the other side of the obstacle, we select a point C at a location suitable for GNSS and leveling measurements. If the ratio $\Delta\zeta_{AB}/S_{AB}$ in the direction A-B is determined, it can be used to determine the analogous ratio between points B and C. In this case, point C placed on the opposite bank of the river may be a benchmark of the leveling network or an auxiliary benchmark. Point C, similarly as in the previous method, must be linked to other benchmarks in the network.

In this method, GNSS measurement is performed on benchmarks A, B, and C, and as already mentioned, a leveling must be performed between benchmarks A and B, as well as the connection of a benchmark C to D. On the basis of relation (1) applied to the segments A-B and B-C, the formula (6) for the normal height difference ΔH in this method may be formulated:

$$\Delta H_{BC} = \Delta h_{BC} - \Delta\zeta_{AB} \frac{S_{BC}}{S_{AB}} = \Delta h_{BC} - (\Delta h_{AB} - \Delta H_{AB}) \frac{S_{BC}}{S_{AB}} \tag{6}$$

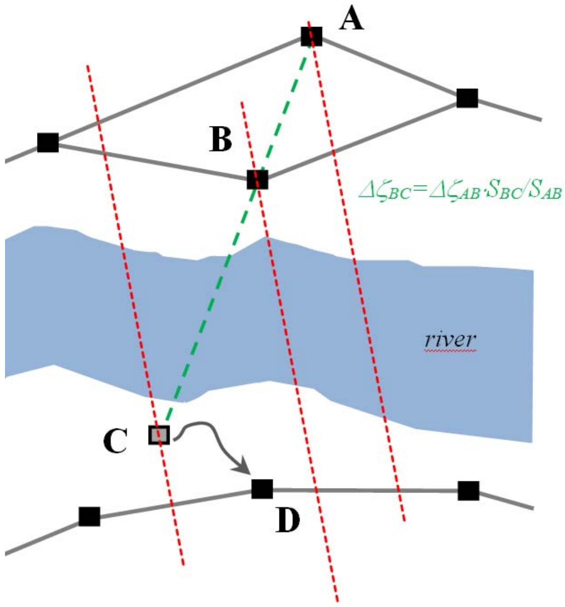


Fig. 4. Outline of the use of information on the course of quasigeoid in leveling through field obstacles in method II

Table 1 shows a numerical example corresponding to the outline in Figure 4 and the relation (6):

Table 1. Computation of height difference in the method (II) – an example

Segment	S [m]	Δh [m]	Δζ [m]	ΔH [m]
A-B	350	2.000	-0.016	2.016
B-C	420	4.000	-0.020	**4.020**

The above method of using quasigeoid in leveling through field obstacles assumes that the quasigeoid course is uniform in the leveled area. This condition is met in small areas (e.g., within a radius of several hundred meters), with undiversified terrain relief. The accuracy of determination of height difference Δh_{BC}, similarly as previously, shall be adjusted to the expected accuracy of the height difference from classical leveling.

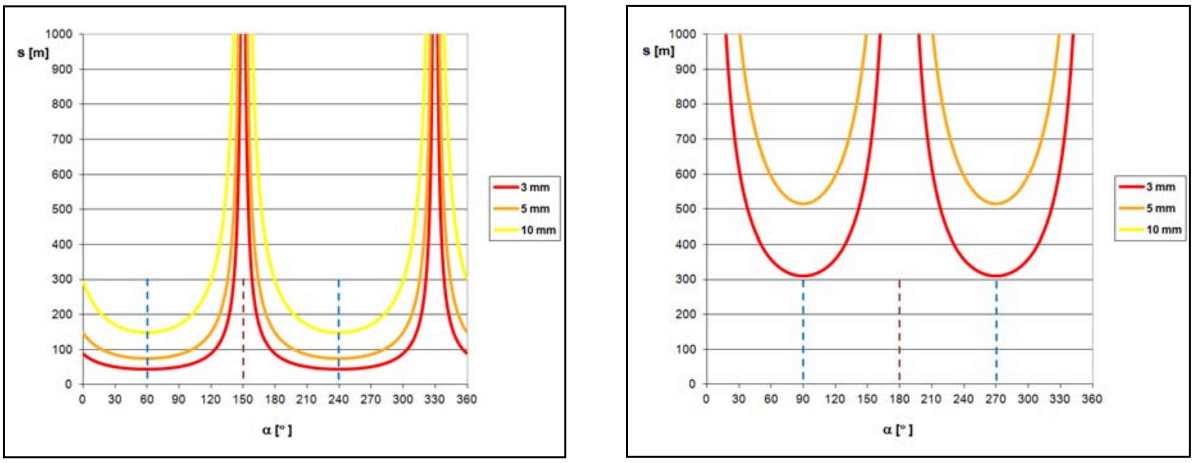
4. Varied conditions for using quasigeoid in leveling

According to the relationship (2), there exists a relationship between the direction of quasigeoid to ellipsoid slope (α), and the difference between height differences from classical and GNSS leveling i.e., the increment of height anomaly ($\Delta\zeta$) and the length of the leveling segment (*s*). This relation may be analyzed for selected areas in Poland, characterized by various quasigeoid to ellipsoid slope.

Graphs in Fig. 5 show the increment value $\Delta\zeta$ in relation to the azimuth of the leveling section and its length, for two places in Poland. The first graph (Fig. 5a) was developed for a region where the components of the slope are very large and amount to: $\xi=7''$ i $\eta=12''$[2] (Krasnystaw surroundings, lubelskie province). The direction of maximum quasigeoid to ellipsoid slope is $\alpha_{\Theta max} = 60°$ in this place. In order to achieve the agreement of results from classic leveling and GNSS leveling on the level of 3 mm (red line on the graph), leveling segments of no more than 50 m would

[2] Values computed on the basis of the model *Geoida niwelacyjna 2001* (Pażus et al., 2002)

be required in this direction. Agreement on the level of 1 cm (yellow line) would be obtained for segments no longer than 150 m. On the other hand, in a direction close to the minimum slope $\alpha_{\Theta min}$ = 150°, this agreement will be obtained for very long segments. For example, for a segment with an azimuth α=120° agreement of results for both types of leveling on the level of 1cm will be obtained for distances no longer than 300m.



a) b)

Fig. 5. Value of a height anomaly increment $\Delta\zeta$ in reference to the azimuth and length of a leveling segment:
a) in an area with a large quasigeoid to ellipsoid slope,
b) in an area with minimum quasigeoid to ellipsoid slope

The second graph (Fig. 5b) was developed for a place where the slope of the quasigeoid to ellipsoid is slight, because its components are ξ=0″ and η=2″ (Malbork surroundings, Pomeranian province). The azimuth of maximum slope in this area is $\alpha_{\Theta max}$ = 90°. In this region, agreement between classical and GNSS leveling will be achieved for much longer leveling segments than before. In the direction of maximum slope ($\alpha_{\Theta max}$ = 90°) the agreement on the level of 3 mm will be obtained for segments no longer than 300 m. On the other hand, 1 cm agreement will be achieved for segments longer than 1 km, regardless of their azimuths.

Comparing both analyzed areas, it can be stated that in the second one replacing classical leveling with GNSS leveling may be much easier to implement. The direction of a leveling segment is not that important, and therefore the replacement of classical differential leveling with GNSS leveling may be done for longer measurement segments. It should be emphasized that in Poland, despite a fairly regular course of quasigeoid with respect to the ellipsoid, the slope of these surfaces varies. This influences possibilities of using GNSS leveling technique in leveling works.

In Poland, the quasigeoid to ellipsoid slope varies from 0.5mm/km to 115mm/km. Therefore, the application of the above described methods should be preceded by the analysis of quasigeoid course in the area where leveling is performed and the location of an auxiliary point on the opposite side of the field obstacle should be adapted to local conditions describing a course of the quasigeoid.

5. Summary

Two methods of overcoming obstacles whilst performing leveling works proposed in this paper indicate the importance of GNSS measurements to accomplish this goal. However, the conditions of their practical application depend on the location within the country where the measurements are performed. Considerations indicate the need for analyzing variation of the quasigeoid to ellipsoid slope across the country. It is necessary to obtain information on where and in which azimuths it is possible and technically justified to replace the classical leveling with GNSS satellite leveling. The analysis of changes in quasigeoid to ellipsoid slope can be made on the basis of the aforementioned national quasigeoid models, calculating the slope components ξ, η. This will allow to present practical recommendations for location of intermediate benchmarks in the aforementioned leveling methods through obstacles.

Cartograms generated on the basis of an analysis of the quasigeoid to ellipsoid slope could facilitate a decision on replacing classical leveling with an appropriately conducted GNSS measurement, in particular when leveling through terrain obstacles.

It is worth emphasizing that as a result of modernization of ASG-EUPOS network in 2017, it is planned to thicken the network of reference stations in some parts of Poland, which will allow for a better coverage of the country with correction data from the GPS and GLONASS systems and will enable the use of RTK and RTN correction data in height measurements at a higher level of accuracy.

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