DE GRUYTER OPEN

Reports on Geodesy and Geoinformatics vol. 96 /2014; pages 1-8 DOI: 10.2478/rgg-2014-0001

DETERMINATION OF SHIP APPROACH PARAMETERS IN THE POLAR COORDINATES SYSTEM

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Abstract

An essential aspect of the safety of navigation is avoiding collisions with other vessels and natural or manmade navigational obstructions. To solve this kind of problem the navigator relies on automatic anti-collision ARPA systems, or uses a geometric method and makes radar plots. In both cases radar measurements are made: bearing (or relative bearing) on the target position and distance, both naturally expressed in the polar coordinates system originating at the radar antenna. We first convert original measurements to an ortho-Cartesian coordinate system. Then we solve collision avoiding problems in rectangular planar coordinates, and the results are transformed to the polar coordinate system. This article presents a method for an analysis of a collision situation at sea performed directly in the polar coordinate system. This approach enables a simpler geometric interpretation of a collision situation.

Keywords: safety of navigation, maritime navigation, navigational mathematics, ARPA, EPA

1. Introduction

Navigation is a process of conducting a ship along a trajectory predetermined in a voyage or passage plan. The ship's tasks are performed by satisfying several restrictions: cost-effectiveness, time, geometric, hydrometeorological, safety of people, cargo and environment and others. The process may be presented as a group of activities (Banachowicz, 2000):

- nautical data acquisition and processing,
- navigational planning,

- determination of ship's speed vectors (over ground, through water and relative to other ships),
- position determination (current position relative to the planned trajectory),
- monitoring positions of relative navigational dangers (target ships, dangerous depth contour, isolated dangers, fairways, traffic separation schemes, prohibited areas and others),
- decision making,
- steering the ship,
- dynamic alteration of the trajectory.

The navigator, officer of the watch, or automated navigational systems, have a number of navigational problems to be solved at sea, including identification of close quarters situation and collision avoiding manoeuvre (Banachowicz & Wołejsza, 2008a). The problem is most often solved using radiolocation systems (navigational radars, ARPA or EPA) (Danish Maritime Administration). Now that the AIS system has become fairly common on sea-going ships, the determination of navigational parameters can be more precise and automatically exchanged between ships or ship and shore when the ship is proceeding in an AIS-covered area.

This article presents possible applications of the polar coordinate systems fixed with own ship for calculations of ship encounter parameters and anti-collision manoeuvres. Our considerations are illustrated with a simulated navigational situation.

2. Encounter parameters calculated in anti-collision systems

Based on radar measurements of target's position (bearings and range) or using AIS data (position and speed vector of target ship), we can determine relative positions of ships and a collision situation. The latter is defined by approach parameters. These parameters are a Closest Point of Approach (CPA) and Time to Closest Point of Approach (TCPA). Solving a problem of anti-collision manoeuvre, these parameters are calculated from these relations (Banachowicz & Wołejsza, 2008a; Wawruch, 1994):

- CPA

$$CPA = \frac{X \cdot \left|V_{w(Y)}\right| - Y \cdot \left|V_{w(X)}\right|}{V_w}, \tag{1}$$

where:

X, Y – distances between ships along the axes x and y,
V_w – relative speed,
$V_{w(X)}$, $V_{w(Y)}$ – components of relative speed,

- TCPA

$$TCPA = \frac{\sqrt{R^2 - CPA^2}}{V_w}, \tag{2}$$

where R – distance between ships.

A solution to a collision situation is reached by calculating a safe course made good of own ship, calculated from this equation:

$$KDd = K_w - 180^0 - \beta = NR + 180^0 + \gamma - 180^0 - \beta = NR + \gamma - \beta =$$
$$= \operatorname{arctg}\frac{Y}{X} + \arcsin\frac{CPA_{LIMIT}}{R} - \arcsin\left(\frac{V_{TG}}{V_{OS}}\sin\alpha\right), \quad (3)$$
$$\alpha = K_{TG} - K_w$$

where:

- X, Y – target ship's coordinates in the local system fixed with own ship,
- NR – true bearing on target ship,
- CPA_{LIMIT} – assumed safe distance,
- V_{TG} – speed of target ship,
- V_{OS} – speed of own ship,
- α, β, γ –auxiliary angles.

The symbols used are illustrated in the drawing below (Fig. 1).

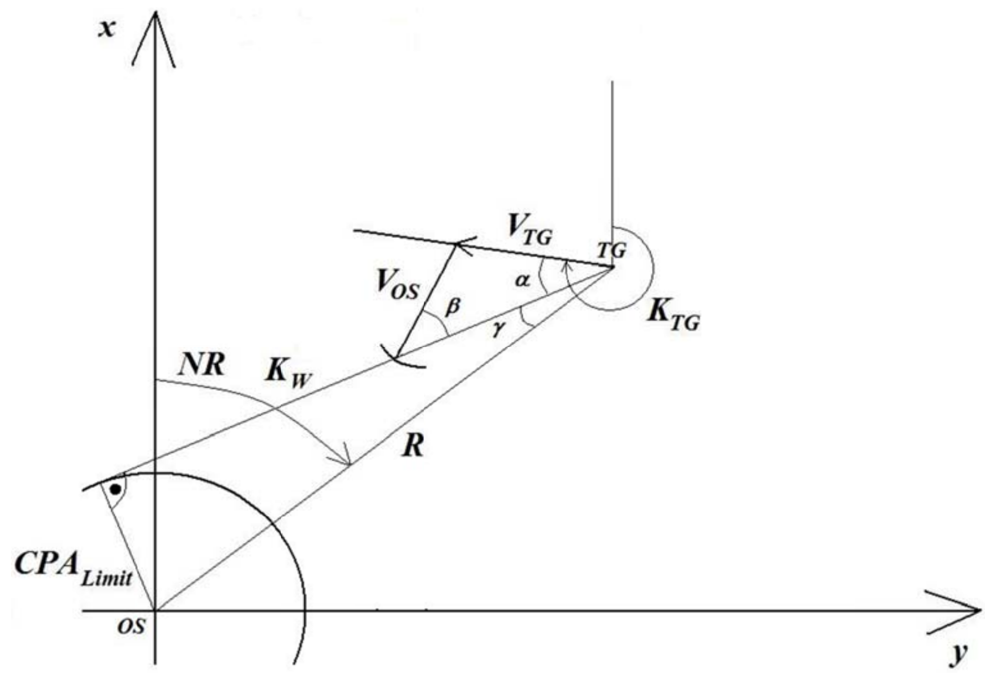

Fig. 1. Primary and auxiliary navigational parameters.

3. Planar coordinate systems

In case of anti-collision manoeuvres, we can use a plane locally tangent to the earth ellipsoid as a reference plane, as well as two coordinate systems fixed to own ship: ortho-Cartesian $0xy$ and polar $0\rho\varphi$. The interrelation between the two systems is described by the equations below:

$$x = \rho \cdot \cos\varphi, \quad y = \rho \cdot \sin\varphi,$$
$$\rho = \sqrt{x^2 + y^2}, \quad \sin\varphi = \frac{y}{\sqrt{x^2 + y^2}}. \quad (4)$$

The relative trajectory of both ships – own ship (OS) and target (TG) is a straight line that placed in ortho-Cartesian coordinates is described by the equation (Fig. 2).

$$y = ax + b. \tag{5}$$

The same trajectory in polar coordinates is described by the equation (Fig. 3). In this case ρ is interpreted as a distance to the target, φ is the bearing on the target from own ship.

$$\rho = \frac{b}{\sin\varphi - a \cdot \cos\varphi}. \tag{6}$$

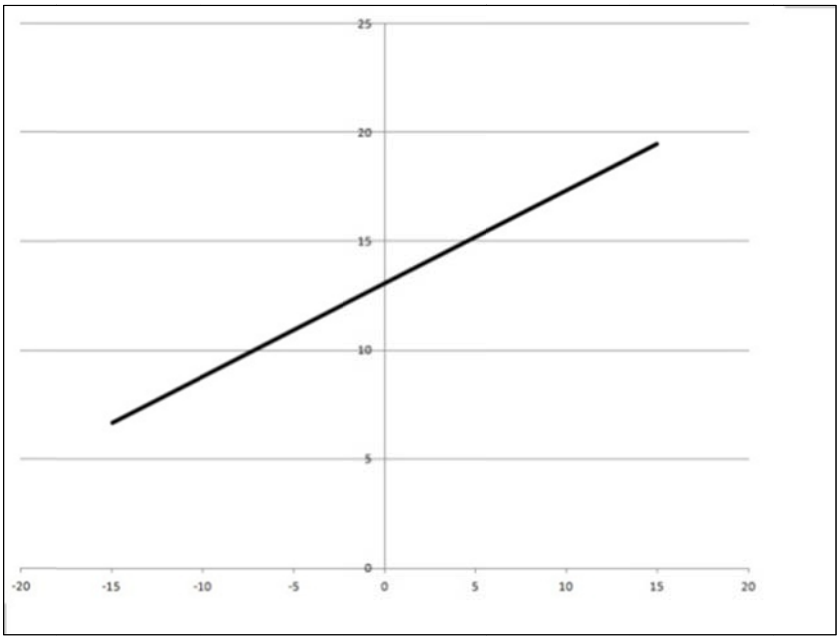

Fig. 2. A relative trajectory in planar ortho-Cartesian coordinates.

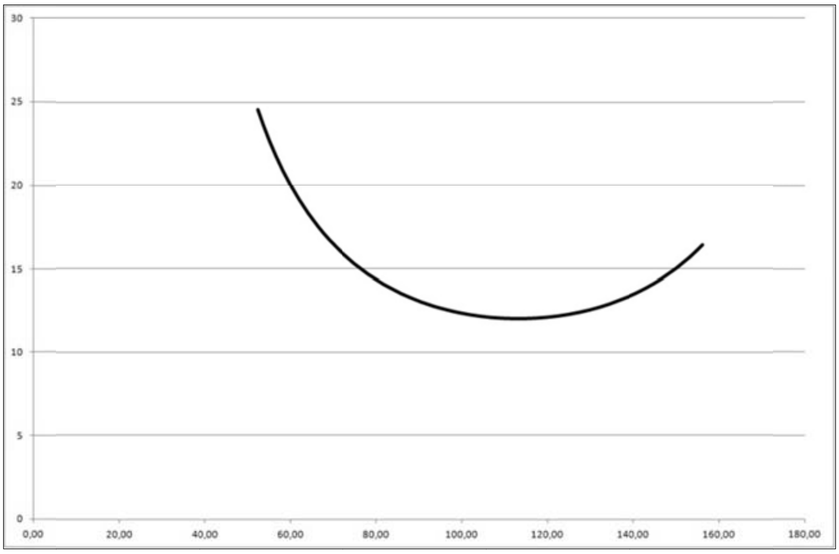

Fig. 3. A straight line on a plane in the polar coordinates system.

4. Ship encounter parameters in polar coordinates

By ortho-Cartesian-to-polar coordinates conversion we can solve an anti-collision problem with a better insight into the situation. That is because in the polar system we use the same type of parameters as measured by radars, i.e. bearing and distance.
The parameters of the relative trajectory in the polar coordinates are described by the relations below. Coefficients a and b represent the formulas:

$$a = \frac{\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1}{\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1}, \tag{7}$$

$$b = \rho_1(\sin\varphi_1 - a \cdot \cos\varphi_1) = \rho_2(\sin\varphi_2 - a \cdot \cos\varphi_2) \tag{8}$$

and

$$b = \rho_1\left(\sin\varphi_1 - \frac{\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1}{\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1} \cdot \cos\varphi_1\right) = \\ = \rho_2\left(\sin\varphi_2 - \frac{\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1}{\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1} \cdot \cos\varphi_2\right). \tag{9}$$

Hence the equation of the relative trajectory in the polar coordinates, after substituting (7) and (9) into (6) has this form

$$\rho = \frac{\rho_1\left(\sin\varphi_1 - \frac{\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1}{\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1} \cdot \cos\varphi_1\right)}{\sin\varphi - \frac{\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1}{\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1} \cdot \cos\varphi}. \tag{10}$$

Let us introduce an Euclidean distance between two point on a plane

$$d_{xy} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

After transformation into polar coordinates, we obtain

$$d_{\rho\varphi} = \sqrt{(\rho_2 \cdot \cos\varphi_2 - \rho_1 \cdot \cos\varphi_1)^2 + (\rho_2 \cdot \sin\varphi_2 - \rho_1 \cdot \sin\varphi_1)^2}.$$

We furtherget

$$d_{\rho\varphi} = \sqrt{\rho_2^2\cos^2\varphi_2 - 2\rho_1\rho_2\cos\varphi_1\cos\varphi_2 + \rho_1^2\cos^2\varphi_1 + \rho_2^2\sin^2\varphi_2 - 2\rho_1\rho_2\sin\varphi_1\sin\varphi_2 + \rho_1^2\sin^2\varphi_1}$$

and after appropriate transformations

$$d_{\rho\varphi} = \sqrt{\rho_2^2(\sin^2\varphi_2 + \cos^2\varphi_2) + \rho_1^2(\sin^2\varphi_1 + \cos^2\varphi_1) - 2\rho_1\rho_2(\cos\varphi_1\cos\varphi_2 + \sin\varphi_1\sin\varphi_2)}$$

$$d_{\rho\varphi} = \sqrt{\rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\varphi_1 - \varphi_2)}. \tag{11}$$

We have obtained a relative track covered by a target as a function of polar coordinates. This is a known the cosine formula, or Carnot's theorem.
The minimum distance ρ_{min} and corresponding angle φ_{min} is calculated as a minimum of the relative trajectory in the polar coordinates (6). Let us calculate a necessary condition for the existence of an extremum (minimum in this case), that is the first derivative of function (6) which we compare to zero. The condition

$$\frac{d\rho}{d\varphi} = -\frac{b(\cos\varphi + a\cdot\sin\varphi)}{(\sin\varphi - a\cdot\cos\varphi)^2} = 0$$

is fulfilled when, and only when

$$b = 0 \text{ or } \cos\varphi + a\cdot\sin\varphi = 0. \quad (12)$$

In the former case the relative trajectory passes through own ship (origin of the coordinate system), which means a collision. Let us then consider the latter case. If we divide equation (12) on both sides by $\cos\varphi$, after transformations we get

$$\varphi_{min} = \text{arc tg}\left(-\frac{1}{a}\right). \quad (13)$$

Hence, for φ_{min} (13) we will obtain a minimum distance

$$\rho_{min} = \frac{b}{\sin\varphi_{min} - a\cdot\cos\varphi_{min}}. \quad (14)$$

Eventually, we will get $CPA = \rho_{min}$ or $CPA = 0$ (for $b = 0$).
The relative speed equals

$$V_\rho = \frac{d_{\rho\varphi}}{t_2 - t_1}, \quad (15)$$

while the time to closest point of approach is expressed by this relation

$$TCPA = t_2 + \frac{d_{\rho\varphi_{min}}}{V_\rho}. \quad (16)$$

5. An example

Let us assume that: at 10:20 hours the first bearing 100^0 on a target was taken at a distance of 8 nautical miles, and at 10:40 another bearing 30^0 and distance 4 nautical miles were measured (Fig. 4).

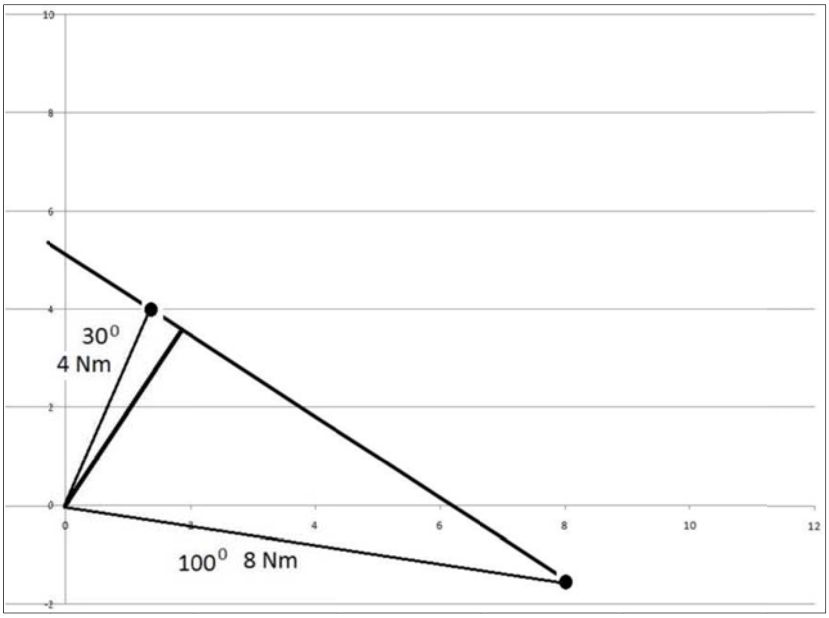

Fig. 4. A relative trajectory of a target in ortho-Cartesian coordinates.

The target's relative track $d_{\rho\varphi} = 7.62$ Nm, its relative speed $V_{\rho} = 22.87$ knots. The calculated approach parameters are $CPA = 3.94$ Nm, bearing 39.54^0 (Fig. 5), $TCPA$ 11:00.

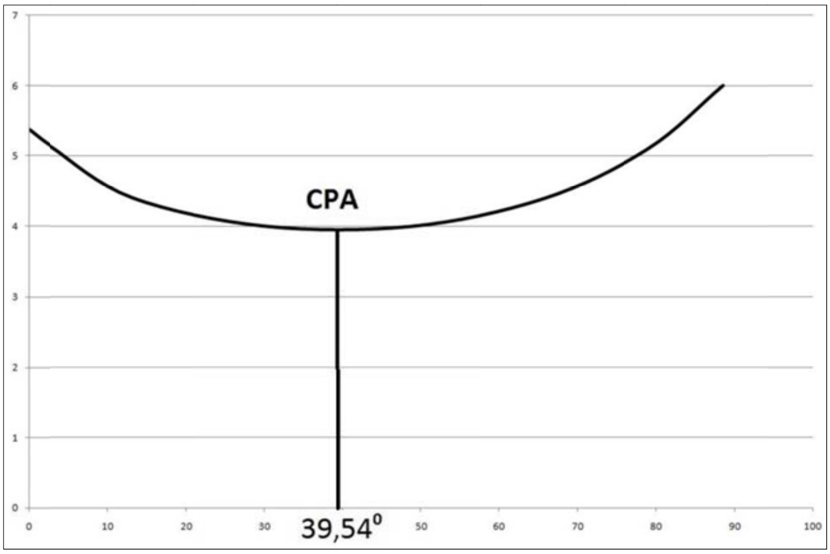

Fig. 5. Relative trajectory of a target in polar coordinates.

6. Conclusions

Problems of collision avoidance at sea are among the most vital issues of safe navigation. Parties concerned do not spare time and financial effort to enhance the safety of shipping. In terms of law, these issues are regulated by the Collision Regulations, International Convention on Safety of Life at Sea, and the International Convention on Standards of Training, Certification and Watchkeeping for Seafarers.

Modern ships have better navigational equipment and systems, including collision prevention systems. Formal qualifications of personnel are also high. All navigating officers have to have at least a secondary general education plus proper professional competences, including radar and ARPA operator certificates. Despite all this, collisions still occur, and quite frequently. One reason is overconfidence in the information received from anti-collision systems or improper interpretation of system indications. Consequences of wrong judgment of a collision situation are disastrous not only in poor visibility conditions. Post-accident analyses (Banachowicz & Wołejsza, 2008a), (Danish Maritime Administration) show that researchers should continue to develop measurement and calculation methods, equipment and methods of visualization and interpretation of a collision situation.

The proposed herein method of calculations of a collision situation is equivalent to known analytical methods (Wawruch, 1994). Producing the same numerical results, it has one essential advantage: it can be visualized. In the relative trajectory displayed in the polar coordinates system one can see instantly the minimum of the trajectory as a function. A similar idea of display is utilized in aircraft observation radars: air traffic control or anti-aircraft artillery.

References

Banachowicz, A. (2000). *Planowanie podróży w ECDIS.* XV Konferencja Katedr i Zakładów Geodezji na Wydziałach Niegeodezyjnych „Metody Geodezji, Fotogrametrii i Teledetekcji dla Inżynierii Środowiska i Budownictwa, Warszawa, 25-26 września 2000.

Banachowicz, A. & Wołejsza, P. (2008a). The Analysis of Possibilities How the Collision Between mv “Gdynia” and mv “Fu Shan Hai” Could Have Been Avoided. TRANSNAV – *International Journal on Marine Navigation and Safety of Sea Transportation*, Vol. 2, No 4, pp. 377-381.

Banachowicz, A. & Wołejsza, P. (2008b). Calculation Accuracy of Safe Course Made Good in an Anticollision System. *Transport Problems – International Scientific Journal,* No 2, Vol. 3.

Danish Maritime Administration. *Casualty investigation reports*. www.dma.dk

Wawruch, R. (1994). *Urządzenie do automatycznego prowadzenia nakresów radarowych (ARPA)*. WSM, Gdynia.

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