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NEW METHOD FOR DETERMINATION OF ADJUSTMENT CORRECTIONS FOR CRANE RAIL AXES

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Abstract

Electronic tacheometers are currently a standard instrument used in geodetic work, including also geodetic engineering measurements. One of the many applications of tacheometers in engineering geodesy are 3D control measurements of crane rail axes. This paper proposes a new method of computing adjustment corrections for crane rail axes based on 3D polar measurements performed with an electronic tacheometer. The intermediary method with conditions on parameters was used in the solution of the problem. The theoretical discussion was complemented with an example application on simulated results of observations. The obtained results confirmed the theoretical assumptions and encourage the verification of the presented proposal on practical examples.

Keywords: crane rail, 3D control measurements, adjustment with conditions

1. Introduction

The high measurement accuracies of modern electronic tacheometers have made them standard equipment applied in geodetic measurements. The use of modern tacheometers in engineering geodesy, including mainly the monitoring of engineering buildings and structures, control measurements as well as measurements of displacements and strains is of great importance. Electronic tacheometers enable the determination of the coordinates of controlled points in a uniform, local 3D coordinate system using modern solutions (e.g. the ATR system) as well as specialized software. One of the measurements of this type are measurements connected with the installation and control of crane rails. The geometric conditions necessary for the correct work of a crane are checked in the scope of these problems (e.g.: Grala, Kopiejewski 2003, Kovanič et al. 2010, Osada 2002, Multi-author work, Engineering

Geodesy Vol. II). In addition to the above-mentioned geometric conditions, maintenance of the proper shapes, dimensions and mutual positions of its components by a crane is also necessary for the correct work of the crane. These problems are presented in the literature of the subject, among others, by Bryś et al. (2009), Gocał (2010), Janusz (1993), Engineering Geodesy Vol. II. The technology of measurements and analysis of their results using the 3D polar method carried out with electronic tacheometers is described by the paper Gocał (2010).

The application of such highly accurate equipment in measurements enables the application of new or modification of known methods for the analysis of the results of observations. This paper proposes the use of an intermediary method with conditions on parameters in the computation of adjustment corrections. The subsequent part of the paper presents theoretical discussion, which is then complemented with an example of practical application.

2. The parametric method with conditions on parameters

Detailed discussion on this adjustment method is found, among others, in the papers Baran (1999), Wiśniewski (2005). This paper will present only the basic assumptions of this manner of adjustment, necessary in further discussion. Let us further assume that the following system of equations can be created in the undertaken problem:

$$\left.\begin{aligned}\mathbf{AX}+\mathbf{l}=\mathbf{v}\\ \mathbf{BX}+\mathbf{\Omega}=\mathbf{0}\end{aligned}\right\} \tag{1}$$

In the first equation: A – is the known coefficient matrix, X – the vector of estimated parameters, v = $[v_1,...,v_n]$ the vector of corrections, l - the vector of measurement results (absolute terms). The second equation with the form BX $+\mathbf{\Omega}$=0 results from the conditions imposed on the estimated parameters. The conditions can result e.g. from geometric relationships or other adopted criteria. In the proposed solution, we will consider two conditions. One results from the geometry of the crane rails, i.e. the assumed spacing *c* between the axes of both rails. The second condition enables setting one height for both rails and the introduction of appropriate adjustment corrections due to this.

The optimization problem of the parametric method with conditions on parameters has the form:

$$v^TPv = \min \tag{2}$$

where the weight matrix P = diag (p_1, …, p_n); the weight $p_i = 1/m_i^2$; (m_i^2 - the square of mean error of *i* coordinate).

Because the vector X must also satisfy the system of conditional equations, the Lagrangian method can be used to solve the problem (2) in the form:

$$\xi = v^TPv - 2k^T(BX + \mathbf{\Omega}) = 0 \tag{3}$$

where k – vector of correlates.

Searching for the minimum of the function (3), we determine the derivative of the function ξ in relation to X, hence:

$$2\mathbf{v}^T\mathbf{PA} - 2\mathbf{k}^T\mathbf{B} = \mathbf{0} \tag{4}$$

after taking into account that AX+L=v, we obtain:

$$\mathbf{A}^{\mathrm{T}}\mathbf{PAX} + \mathbf{A}^{\mathrm{T}}\mathbf{PL} = \mathbf{B}^{\mathrm{T}}\mathbf{k} \tag{5}$$

The vector X is obtained from the following relationship:

$$\mathbf{X} = -\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PL} - \mathbf{B}^{\mathrm{T}}\mathbf{k}\right) \tag{6}$$

Because the vector X must also satisfy the conditional equations, we will obtain after substitution of the relationship (6) into the relationship BX $+\mathbf{\Omega}$=0 a system of equations of normal correlates in the form:

$$-\mathbf{B}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PL} - \mathbf{B}^{\mathrm{T}}\mathbf{k}\right) + \mathbf{\Omega} = \mathbf{0} \tag{7}$$

Determining the vector of correlates, we will obtain:

$$\mathbf{k} = -\left\{\mathbf{B}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{B}^{\mathrm{T}}\right\}^{-1}\left\{\mathbf{\Omega} - \mathbf{B}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PL}\right\} \tag{8}$$

Computations using the intermediary method with conditions on parameters proceed in the three following stages:

1. $\mathbf{k} = -\left\{\mathbf{B}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{B}^{\mathrm{T}}\right\}^{-1}\left\{\mathbf{\Omega} - \mathbf{B}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{PL}\right\}$
2. $\mathbf{X} = -\left(\mathbf{A}^{\mathrm{T}}\mathbf{PA}\right)^{-1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{PL} - \mathbf{B}^{\mathrm{T}}\mathbf{k}\right)$
3. v = AX+L

The problem (1) can also be solved by the simplified method (e.g. Wiśniewski 2005). In this solution, the conditional equations BX $+\mathbf{\Omega}$=0 can be used as the equations of corrections with high values of weights, hence:

$$\mathbf{BX} + \mathbf{\Omega} = \mathbf{v}_{\Omega} \text{ with weights } \mathbf{P}_{\infty} \tag{9}$$

where $\mathbf{P}_{\infty} = \mathrm{diag}(\eta_1,...,\eta_r);\ \ \eta$ - any number, *r* – number of conditions.

The correctness condition for the computations is the adoption of such high weights that the corrections obtained from the solution are equal to zero (within the accuracy of the conducted computations).

Computations in this way are carried out by the parametric method, in which the system of equations of corrections has the following form:

$$\tilde{\mathbf{A}}\mathbf{X} + \tilde{\mathbf{L}} = \tilde{\mathbf{v}} \tag{10}$$

where:

$$\tilde{\mathbf{A}} = \begin{bmatrix}\mathbf{A}\\ \mathbf{B}\end{bmatrix},\quad \tilde{\mathbf{L}} = \begin{bmatrix}\mathbf{L}\\ \mathbf{\Omega}\end{bmatrix},\quad \tilde{\mathbf{v}} = \begin{bmatrix}\mathbf{v}\\ \mathbf{v}_{\Omega}\end{bmatrix} \tag{11}$$

with weights:

$$\tilde{\mathbf{P}} = \begin{bmatrix}\mathbf{P} & \mathbf{0}\\ \mathbf{0} & \mathbf{P}_{\infty}\end{bmatrix} \tag{12}$$

Using the solution by the parametric method, we obtain:

$$\mathbf{X} = -\left(\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{A}}\right)^{-1}\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{L}} \tag{13}$$

Hence, the vector:

$$\tilde{\mathbf{v}} = -\tilde{\mathbf{A}}^{\mathrm{T}}\left(\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{A}}\right)^{-1}\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{L}} + \tilde{\mathbf{L}} \tag{14}$$

Statistical interpretation of estimation results consists in the determination, among others, of variance-covariance matrices of adjusted results of the measurement vectors X as well as the vectors of corrections v.
The covariance matrix of parameters C_X has the following form:

$$\mathbf{C}_{\mathbf{X}} = m_0^2\mathbf{Q}_{\mathbf{X}} \tag{15}$$

in which $\mathbf{Q}_{\mathbf{X}}$ is the cofactor matrix with the form:

$$\mathbf{Q}_{\mathbf{X}} = (\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{A}})^{-1} \tag{16}$$

The covariance matrix of the vector of corrections C_V is equal to:

$$\mathbf{C}_{\mathrm{v}} = m_0^2\mathbf{Q}_{\mathrm{v}} \tag{17}$$

with the cofactor matrix:

$$\mathbf{Q}_{\mathrm{v}} = (\tilde{\mathbf{P}} - \tilde{\mathbf{A}}(\tilde{\mathbf{A}}^{\mathrm{T}}\tilde{\mathbf{P}}\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}^{\mathrm{T}}) \tag{18}$$

We will determine the coefficient m_0^2 from the following relationship:

$$m_0^2 = \frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{n-m} \tag{19}$$

3. Observation equations

Let us assume that when performing geodetic measurements of rail axes with an electronic tacheometer, the coordinates of the controlled points *x*, *y*, *z* were determined in a local 3D system. The appropriate system of observation equations should be used to perform the computations. The axes of both crane rails are parallel lines satisfying the equations:

- left rail

$$y=ax+b \tag{20}$$

- right rail

$$y=ax+b+c \tag{21}$$

The heights of both rails should be the same (within the tolerance specified by the appropriate standard) and be:

$$z=z^{w} \tag{22}$$

Due to this, the equations of corrections will assume the following form:

- left rail (horizontal adjustment)

$$v_{y_i} = ax_i + b - y_i \tag{23}$$

- right rail (horizontal adjustment)

$$v_{y_i} = ax_i + b + c - y_i \tag{24}$$

- left and right rail (height adjustment)

$$v_{z_i} = z^w - z_i \tag{25}$$

where: v_{y_i}, v_{z_i} - are corrections to observations (in this case of coordinates *y*, *z*), *a*, *b*, *c* – are the estimated parameters, x_i, y_i, z_i – space coordinates of the controlled points, i= 1,…, *n* – point number: *n* - number of the controlled points.

4. Example of practical application

The practical properties of the proposed method were analysed on a theoretical example, in which simulated results of observations were used. It was assumed that the points 1, 2, 3, 4, 5, 6, 7, 8 (Fig. 1) were points located on the axes of crane rails. The position of the points was defined by space coordinates (Table 1) in a local coordinate system x, y, z.

Table 1. 3D coordinates of the controlled points.

Point no.	*x* [mm]	*y* [mm]	*z* [mm]	Point no.	*x* [mm]	*y* [mm]	*z* [mm]
1	0	0	0	5	20	9985	20
2	3888	-15	20	6	3988	10023	15
3	7998	18	-15	7	8010	9988	-15
4	11997	-8	10	8	12006	9996	-10

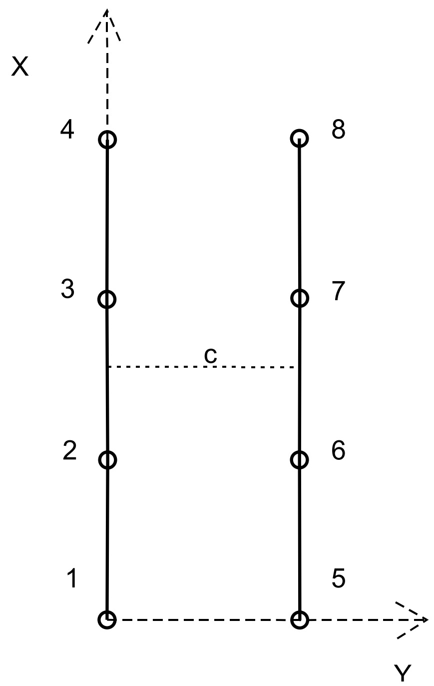


Fig. 1. Distribution of the controlled points (xy system)

Analyses were carried out in two variants using the simplified way of performing computations described above.

Variant 1:
Using the relationships (23), (24), (25), appropriate equations of corrections were formulated for all the control points (*i*= 1, 2, 3, 4, 5, 6, 7, 8).
Hence,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 3888 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 7998 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 11997 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 20 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3988 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 8010 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12006 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} a \\ b \\ c \\ z^w \end{bmatrix} \quad \mathbf{l} = \begin{bmatrix} 0 \\ 0 \\ 15 \\ -20 \\ -18 \\ 15 \\ 8 \\ -10 \\ -9085 \\ -20 \\ -10023 \\ -15 \\ -9988 \\ 15 \\ -9996 \\ 10 \end{bmatrix}$$

One condition was also adopted, concerning the spacing between the axes of both rails in the form of *c*=10 000 mm (e.g. resulting from the technical design). This constraint allowed us to create a condition with the form:

$$c - 10\,000 = 0$$

or, in matrix notation:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ z^w \end{bmatrix} + \begin{bmatrix} -10\,000 \end{bmatrix} = v_\Omega$$

$$\mathrm{B} \quad \mathrm{X} + \mathbf{\Omega} = \mathbf{v}_\Omega$$

Performing the computations, the vector X was determined using the relationship (13) and the vector $\tilde{\mathbf{v}}$ from the relationship (14).

Variant 2:
In variant 2, an additional condition was adopted to the constraint used in variant 1. It was assumed that the height for which adjustment corrections would be computed *z*=10 mm; hence, the restrictive conditions can be written as follows:

$$c - 10\,000 = 0$$
$$z^{W} - 10 = 0$$

or, in matrix notation:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ z^{w} \end{bmatrix} + \begin{bmatrix} -10000 \\ -10 \end{bmatrix} = \begin{bmatrix} v_{\Omega} \\ v_{\Omega} \end{bmatrix}$$

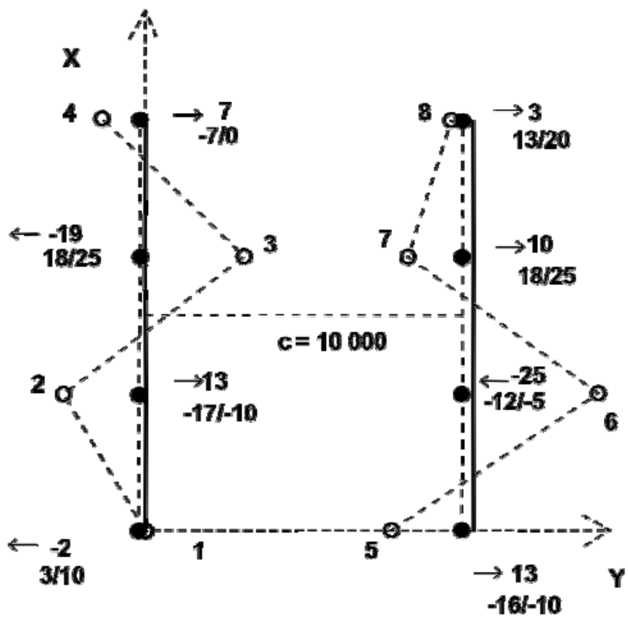
$$\mathrm{B} \quad \mathrm{X} \quad + \quad \boldsymbol{\Omega} \quad = \quad \mathbf{v}_{\Omega}$$

Performing the computations of the variant, the vector of sought parameters X was determined (13) as well as the vector of corrections $\tilde{\mathbf{v}}$ (14). Table 2 compiles the obtained results of computations.

Table 2. Results of computations

Parameters and corrections		Variant 1 [mm]	Variant 2 [mm]
X parameters	a	0	0
	b	-2	-2
	c	10 000	10 000
	z^{w}	3	10
Corrections	v_{Y_1}	-2	-2
	v_{Z_1}	3	10
	v_{Y_2}	13	13
	v_{Z_2}	-17	-10
	v_{Y_3}	-19	-19
	v_{Z_3}	18	25
	v_{Y_4}	7	7
	v_{Z_4}	-7	0
	v_{Y_5}	13	13
	v_{Z_5}	-17	-10
	v_{Y_6}	-25	-25
	v_{Z_6}	-12	-5
	v_{Y_7}	10	10
	v_{Z_7}	18	25
	v_{Y_8}	3	3
	v_{Z_8}	13	20
	v_{C}	0	0
	$v_{Z^{w}}$	------	0

Figure 2 visually presents the drawing of corrections for determination of the axes of both crane rails. The values of corrections for height adjustment are also given.



2 ○ point before adjustment
6 ● point after adjustment
--------- axis after adjustment
- - - - - - - - - axis before adjustment
14 horizontal adjustment correction [mm]
-17/-30 vertical adjustment correction variant 1/variant 2 [mm]
⟵ direction of adjustment correction

Fig. 2. Adjustment corrections in the xy system

5. Summary

This paper proposes the use of the intermediary method with conditions on parameters for analysis of the results of measurements obtained with an electronic tacheometer in a 3D system. The proposed method enables the determination of adjustment corrections for crane axes in a uniform spatial coordinate system. The test presented in the paper is an example of simulated measurement results. Appropriate tests should therefore be performed on actual results of observations. However, it can already be concluded that the obtained results encourage the practical use of the presented data analysis method.

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